Topic Study Group 5

Teaching and Learning of Number and Arithmetic

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1. Aims of the TSG

The purpose of this TSG was to gather congress participants who were interested in research and development in the teaching and the learning of number systems and arithmetic through activities in and out of school. The mathematical domains include whole numbers, integers, ratio and proportion, and rational numbers as well as representations and problem-solving using numbers related to each of these domains:

- *research-based specifications of domain-specific goals,*
- *analysis of learning processes and learning outcomes in domain-specific learning environments*
- *and classroom cultures,*
- *new approaches to the design of meaningful and rich learning environments and assessments.*

We encouraged submissions that offered theoretical or empirical contributions and sought to include research from a variety of cultural contexts to enhance our discussions.

1.1. Submissions

We received 37 submissions from 18 countries (US: 5; Canada: 2; Brazil: 6; China: 1; Japan: 1; UK: 2; Germany: 3; Mexico: 3; South Africa: 4; Spain: 1; France: 1; Australia: 1; Sweden: 2; Isrel:1; Italy: 1; Nigeria: 1; Bruni: 1; Chile: 1), thus reaching our goal of diverse cultural representation. Of those 37 submissions, twenty-nine were accepted as paper presentations, six as posters, and two was rejected.

1.2. Sessions

There were so many high-quality submissions the ICMI organizing committee granted our TSG one more time slot for presentations. In our first 120-minute session, the TSG Chair, Arthur Powell, introduced the rest of the Team and described the format of the sessions. Generally, there are four 20 minutes *long* oral presentations and discussions and 13 *short* oral presentations, and after every 2 or 3 presentations, there were 10 minutes of collective discussions. Throughout the four days, we attempted to facilitate participant

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dialogue in order to collectively identify emerging research themes, potential interdisciplinary approaches, and future research opportunities. Discussion time was critical for fostering participant networking as well as countering "virtual conference" fatigue. At the end of our last session, we scheduled 50 minutes for whole group reflection, discussions, and suggestions for needed research trajectories.

1.3.   Paper topics

Of the 29 accepted papers, only 18 were able to be presented during the online conference. In Tab. 1 below is a list of these papers and authors in order of presentation.

Tab. 1. List of papers presented

Paper and author(s)
[1] Representational flexibility linked to higher attainment in early number learning. ***Samantha Morrison*** (South Africa).
[2] Conceptual and procedural understanding on addition of fractions among Year 5 primary children. ***Nor'Arifahwati Abbas***, *Masitah Shahrill, Mohd Khairul Amilin Tengah, Nor Azura Abdullah* (Brunei).
[3] South African learner's patterns of performance on additive word problems. ***Herman M. Tshesane*** (South Africa).
[4] The case against coherence in mathematics instruction. ***Ola Helenius*** *and Linda Marie Ahl* (Sweden).
[5] Identifying South African primary learners doubling and halving reasoning through a written assessment. ***Sameera Hansa*** *and Hamsa Venkat* (South Africa).
[6] The use of arrays in solving multiplication word problems in Grade 4. ***Mayamiko Malola*** (Australia).
[7] Toward a universal cognitive core: A cross-cultures (USA, China) progression in multiplicative reasoning. ***Ron Tzur*** (invited speaker, USA) *and Rui Ding* (China).
[8] Flexible mental calculation: A study with 2nd and 4th Grade Brazilian students. ***Luciana Vellinho Corso***, *Sula Cristina Teixeira Nunes, and Évelin Fulginiti de Assis* (Brazil).
[9] The flexibility in mental calculation: Characterizing the profiles of a group of Brazilian Elementary Students. ***Évelin Fulginiti de Assis***, *Sula Cristina Teixeira Nunes, and Luciana Vellinho Corso* (Brazil).
[10] Precursors of problem-solving in two Brazilian cities: the role of social and economic differences. ***Beatriz Vargas Dorneles***, *Camila Peres Nogues, and Elielson Magalhães Lima* (Brazil).
[11] The performance in domain-specific cognitive abilities among low and typical mathematical achievers. ***Camila Peres Nogues***, *Elielson Magalhães Lima, and Beatriz Vargas Dorneles* (Brazil).
[12] Improving student knowledge of fraction magnitude in the early grades. ***Arthur Belford Powell*** *and Candell V. Ali* (Israel).
[13] Elementary teacher professional learning to explore and extend nuanced meaning of number. ***Krista Francis***, ***Sharon Friesen***, *Miwa Takeuchi, Armando Paulino Preciado Babb, and Brown* (Canada).
[14] Difficulties of learning the decimal positional numeration (DPN) system: The principle of exchange. ***Daniela Fernandes*** *and Jeanne Koudogbo* (Canada).
[15] Decimal number system in Quebec Mathematics Program and in textbooks: What Knowledge and for which mathematical education. ***Jeanne Koudogbo*** *and Daniela Fernandes* (Canada).
[16] Errors in ratio and proportion: A framework for analysis. ***Özdemir Tiflis*** *and Gwen Ireson* (UK).
[17] School-readiness in mathematics: Development of a screening test for children starting school. ***Andrea Peter-Koop*** (Germany).
[18] Students performance when solving word problems involving fractions. ***Maria T. Sanz***, *Carlos Valenzuleza, Olimpia Figueras, and Bernardo Gómez* (Spain).

2. Conference Themes

Although there was a large variety of research topics presented during the sessions, the majority of the work can be summarized across three themes. First, a few research teams focused on whole number learning. For example, Samantha Morrison's study[1] focused on "representational flexibility linked to higher attainment in early number learning," and Ola Helenius' group[4] presented a case against coherence in mathematics instruction. Second, Tshesane[3] talked about the South African learner's patterns of performance on additive word problems.

A second theme was multiplicative thinking and reasoning. In particular, Tzur and Ding[7] argued that there was a universal cognitive core in multiplicative reasoning cross-culture (USA and China). Furthermore, they used quantitative data to support that Same-Unit Coordination is the screener in students' multiplicative reasoning and place value concept in base ten. Finally, Corso and her group presented two related papers[8,9], one reported on the quantitative results of 2nd and 4th grade Brazilian students' flexible mental calculation, and the other one described the characteristics of the flexibility in mental calculation of a group of Brazilian elementary students.

Finally, several research teams explored the teaching and learning of fractions. Powell discussed how to improve students' knowledge of fraction magnitude in the early grades. Fernandes and Koudogbo[14] studied the difficulties of learning the decimal positional numeration system and talked about the principle of exchange. They also discussed the decimal number system in the Quebec mathematics program and textbooks.

3. Areas for Future Research

On the final day of the conference, the participants discussed potential future research topics and recommended some papers to publish in related journals.